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ELECTROMOTIVE FORCE AND EFFICIENCY RATIO OF ANISOTROPIC OPTICAL THERMOELEMENTS

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We investigate the dependence of the transverse electromotive force and efficiency ratio of an anisotropic thermoelement on the crystallographic orientation in the case where the anisotropy of the coefficients of thermal emf and thermal conductivity are taken into account.

Along with thermoelements that are based on the effect of the anisotropy of the thermal-emf coefficient [1], optical thermoelements based on the anisotropy of the thermal-conductivity coefficient (AOT) are also of great interest; they form the basis for a number of original thermoelectric instruments and devices [2]. Below we consider an AOT that is a plate 1 with specified dimensions a, b, and c (see Fig. 1) made of a material that is considered to be anisotropic with respect to the coefficients of thermal emf $\hat{\alpha}$ and thermal conductivity $\hat{\chi}$. In a laboratory coordinate system (X, Y, Z) turned through the angle φ in the XOY plane with respect to the crystallographic system (X', Y', Z') the tensors $\hat{\alpha}$ and $\hat{\chi}$ have the form

$$\hat{\alpha} = \begin{vmatrix} \alpha_{\parallel}\sin^2\varphi + \alpha_{\perp}\cos^2\varphi & (\alpha_{\parallel} - \alpha_{\perp})\sin\varphi\cos\varphi & 0 \\ (\alpha_{\parallel} + \alpha_{\perp})\sin\varphi\cos\varphi & \alpha_{\parallel}\cos^2\varphi + \alpha_{\perp}\sin^2\varphi & 0 \\ 0 & 0 & \alpha_{\perp} \end{vmatrix},$$

$$\hat{\chi} = \begin{vmatrix} \chi_{\parallel}\sin^2\varphi + \chi_{\perp}\cos^2\varphi & (\chi_{\parallel} - \chi_{\perp})\sin\varphi\cos\varphi & 0 \\ (\chi_{\parallel} - \chi_{\perp})\sin\varphi\cos\varphi & \chi_{\parallel}\cos^2\varphi + \chi_{\perp}\sin^2\varphi & 0 \\ 0 & 0 & \chi_{\perp} \end{vmatrix}. \tag{1}$$

A radiant flux of density q_0 is incident on the upper face of this thermoelement. The lower face is in contact with a thermostat 2 at $T = T_0$. The transverse thermal emf ε is taken off by means of leads 3 located on the end faces of the plate 1. The upper and side faces of the plate are insulated adiabatically by evacuation; edge effects are not taken into consideration ($a = c >> b$) [3]. The wavelength of the incident radiation flux lies within the transparency range of the material of the thermoelement and the thermostat. Passage of a uniform monochromatic flux of density q_0 and wavelength λ through the plate causes the appearance in it of a temperature gradient and the associated transverse thermal emf [3].

We find the temperature distribution from the basic law of heat conduction in the presence of internal heat sources:

$$\frac{\partial T}{\partial t} = -\frac{1}{c_0 d}\sum_{i=1}^{3}\frac{\partial q_i}{\partial x_i} + \frac{q_v}{c_0 d}, \tag{2}$$

where d is the density; c_0 is the specific heat capacity of the thermoelement material; q_v is the quantity of heat liberated by the internal sources in a unit volume per unit time.

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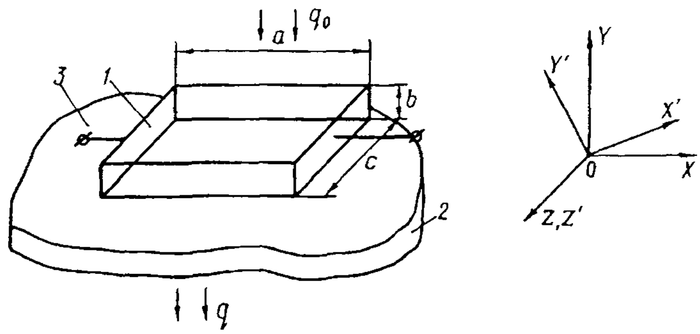

Fig. 1. Anisotropic optical thermoelement: 1) plate made of thermoelectrically anisotropic optically transparent material, 2) removal of heat from the optically transparent heat-conducting material, 3) electrical leads; laboratory system of coordinates XYZ and orientation of the principal crystallographic axes $X'Y'Z'$ of the single-crystal plate.

For a stationary temperature distribution $\partial T/\partial t = 0$, in the approximation $\partial T/\partial x = \partial T/\partial z = 0$, $\chi_{12} < \chi_{22}$, Eq. (2) has the form

$$\chi_{22} \frac{d^2T}{dy^2} + q_v = 0\,. \tag{3}$$

Using the well-known Bouger–Lambert law for q_v, we obtain

$$q_v = q_0 \gamma \exp\left[-\gamma (b - y)\right], \tag{4}$$

where γ is the coefficient of optical absorption of the thermoelement material.

Substitution of Eq. (4) into Eq. (3) yields

$$\chi_{22} \frac{d^2T}{dy^2} + q_0 \gamma \exp\left[-\gamma (b - y)\right] = 0\,. \tag{5}$$

Solving Eq. (5) under the boundary conditions

$$T|_{y=0} = T_0\,, \quad \left.\frac{dT}{dy}\right|_{y=b} = 0\,,$$

we obtain the temperature distribution inside the AOT in the following form:

$$T(y) = T_0 + \frac{q_0}{\chi_{22}} \left\{ y + \frac{\exp(-\gamma b)}{\gamma} \left[1 - \exp(\gamma y)\right] \right\}. \tag{6}$$

The strength of the thermoelectric field E^T is determined by the relation

$$E^T = \hat{\alpha} \nabla T\,. \tag{7}$$

Substituting Eq. (6) into Eq. (7), we obtain

$$E_x^T = \alpha_{12} \frac{dT}{dy} = q_0 \frac{\alpha_{12}}{\chi_{22}} \left\{1 - \exp\left[-\gamma (b - y)\right]\right\}. \tag{8}$$

According to [4], the emf of the AOT is determined by the relation

$$\varepsilon = \frac{1}{bc} \int_0^b dy \int_0^c dz \int_0^a E_x^T \, dx . \tag{9}$$

Substituting Eq. (8) into Eq. (9), with account for Eq. (1), we obtain an expression for the emf of the AOT:

$$\varepsilon = \frac{q_0 a \left(\alpha_{\parallel} - \alpha_{\perp}\right) \sin\varphi \cos\varphi}{\chi_{\parallel} \cos^2\varphi + \chi_{\perp} \sin^2\varphi} \left\{ 1 - \frac{1}{\gamma b} \left[1 - \exp\left(-\gamma b\right) \right] \right\} . \tag{10}$$

Analysis of Eq. (10) shows that in the case of:

a) optical transmission ($\gamma b << 1$)

$$\varepsilon = \frac{q_0 a \left(\alpha_{\parallel} - \alpha_{\perp}\right) \sin\varphi \cos\varphi}{2 \left(\chi_{\parallel} \cos^2\varphi + \chi_{\perp} \sin^2\varphi\right)} \gamma b ; \tag{11}$$

b) bulk absorption ($\gamma b \approx 1$)

$$\varepsilon = \frac{q_0 a \left(\alpha_{\parallel} - \alpha_{\perp}\right) \sin\varphi \cos\varphi}{\exp(1) \left[\chi_{\parallel} \cos^2\varphi + \chi_{\perp} \sin^2\varphi\right]} ; \tag{12}$$

c) surface absorption ($\gamma b >> 1$)

$$\varepsilon = \frac{q_0 a \left(\alpha_{\parallel} - \alpha_{\perp}\right) \sin\varphi \cos\varphi}{\chi_{\parallel} \cos^2\varphi + \chi_{\perp} \sin^2\varphi} . \tag{13}$$

The efficiency of these devices [5] is determined by the relation [6]

$$\eta = \eta_C \frac{1}{1 + \Lambda} , \tag{14}$$

where $\eta_C = (T_1 - T_0)/T_1$ is the Carnot efficiency ratio; $\Lambda = BT_0/A$; A is the work done by these devices.

The rate of entropy production within the volume of the generator is [7]

$$B = \frac{Q_0}{T_0} - \frac{Q_1}{T_1} = \chi_{22} S \left[\frac{1}{T_0} \frac{dT}{dy}\bigg|_{y=0} - \frac{1}{T_1} \frac{dT}{dy}\bigg|_{y=b} \right] , \tag{15}$$

where Q_1 and Q_0 are the heat absorbed from the heater and given to the cooler, respectively; $S = ac$ is the cross section of the upper and lower faces of the AOT; T_1 is the temperature of the upper face.

For the case of the AOT of the problem considered, taking into account Eqs. (6) and (15), we have

$$B = \frac{q_0 ac}{T_0} \left[1 - \exp\left(-\gamma b\right) \right] . \tag{16}$$

We write an expression for the integral current I in the AOT:

$$I = \frac{\varepsilon}{R_i + R_{\text{in}}} = \frac{q_0 a \left(\alpha_{\parallel} - \alpha_{\perp}\right) \sin\varphi \cos\varphi}{\left(\chi_{\parallel} \cos^2\varphi + \chi_{\perp} \sin^2\varphi\right) \left(R_i + R_{\text{in}}\right)} \left\{ 1 - \frac{1}{\gamma b} \left[1 - \exp\left(-\gamma b\right) \right] \right\} , \tag{17}$$

where $R_i = \rho(a/S')$ is the internal resistance of the AOT; $S' = bc$ is the end-face area of the AOT; ρ is the specific resistance of the AOT material.

The work done by the thermogenerator $A = I^2 R_{\text{in}}$, with account for (17) at $R_i = R_{\text{in}}$, is

$$A = \frac{q_0^2 abc (\alpha_{\parallel} - \alpha_{\perp})^2 \sin^2 \varphi \cos^2 \varphi}{4\rho (\chi_{\parallel} \cos^2 \varphi + \chi_{\perp} \sin^2 \varphi)^2} \left\{ 1 - \frac{1}{\gamma b} [1 - \exp(-\gamma b)] \right\}^2 . \tag{18}$$

Then for the dimensionless parameter Λ that enters into the expression for the efficiency factor of the generator, we have

$$\Lambda = \frac{4\rho (\chi_{\parallel} \cos^2 \varphi + \chi_{\perp} \sin^2 \varphi) [1 - \exp(-\gamma b)]}{q_0 b (\alpha_{\parallel} - \alpha_{\perp})^2 \sin^2 \varphi \cos^2 \varphi \left\{ 1 - (\gamma b)^{-1} [1 - \exp(-\gamma b)] \right\}^2} . \tag{19}$$

Substituting Eq. (19) into Eq. (14), with account for Eq. (6), we obtain an expression for the efficiency in the form

$$\eta = \frac{(q_0 b) (\chi_{\parallel} \cos^2 \varphi + \chi_{\perp} \sin^2 \varphi)^{-1} \left\{ 1 + (\gamma b)^{-1} [\exp(-\gamma b) - 1] \right\}}{T_0 + q_0 b (\chi_{\parallel} \cos^2 \varphi + \chi_{\perp} \sin^2 \varphi)^{-1} \left\{ 1 + (\gamma b)^{-1} [\exp(-\gamma b) - 1] \right\}} \times$$

$$\times \left\{ 1 + \frac{4\rho (\chi_{\parallel} \cos^2 \varphi + \chi_{\perp} \sin^2 \varphi)^2 [1 - \exp(-\gamma b)]}{q_0 b (\alpha_{\parallel} - \alpha_{\perp})^2 \sin^2 \varphi \cos^2 \varphi [1 - (\gamma b)^{-1} [1 - \exp(-\gamma b)]]^2} \right\}^{-1} . \tag{20}$$

Analysis of Eq. (20) shows that these AOTs can be used as energy transformers in the case $\gamma b >> 1$, and their use in devices for recording and controlling high-power radiant energy [8] is possible when $\gamma b << 1$.

From expression (10) it is seen that the transverse thermal emf depends on the magnitude of the angle φ. In our case the maximum value of the thermal emf is determined by the anisotropy of the coefficients of thermal emf $\hat{\alpha}$ and thermal conductivity $\hat{\chi}$ and is observed at a certain value of φ. Theoretical and experimental investigations and calculations show that ε_{max} and η_{max} are obtained for CdSb at $\varphi = 35^{\circ}$, for Bi at $\varphi = 38^{\circ}$, and for $ZnAs_2$ at $\varphi = 52^{\circ}$.

It should be noted that the expression for the thermoelectric quality of an AOT will be determined in our case by the regime of its operation:

а) $\gamma b >> 1$

$$Z = \frac{\alpha_{12}^2 \sigma}{\chi_{22}} , \tag{21}$$

б) $\gamma b \approx 1$

$$Z = \frac{\alpha_{12}^2 \sigma}{\chi_{22}} [1 - \exp(-1)] , \tag{22}$$

в) $\gamma b << 1$

$$Z = \frac{\alpha_{12}^2 \sigma}{\chi_{22}} \gamma b . \tag{23}$$

Thus, the magnitude of the thermoelectric quality Z_a, which enters into Eq. (20) and is specified by relations (21)-(23), depends, apart from $\hat{\alpha}$, $\hat{\chi}$, σ, on the optical parameters of the AOT material.

NOTATION

AOT, anisotropic optical thermoelement; $\hat{\alpha}$, thermal-emf coefficient; $\hat{\chi}$, thermal-conductivity coefficient; φ, angle of rotation of the laboratory coordinate system (XYZ) relative to the crystallographic system $(X'Y'Z')$; a, b,

c, geometric dimensions of the anisotropic optical thermoelement; q_0, radiant-flux density; λ, wavelength; T, temperature; d, density; c_0, specific heat capacity of the thermoelement material; q_v, quantity of heat released by internal sources; γ, coefficient of optical absorption; E^T, thermoelectric-field strength; ε, electromotive force; η, efficiency ratio; B, rate of entropy production; A, work; S, area of the working faces of the anisotropic thermoelement; R, resistance; ρ, specific resistance; σ, specific electrical conductivity; Λ, dimensionless parameter; Z, thermoelectric efficiency.

REFERENCES

1. W. Tomson, Math. Phys. Papers, No. 1, 266-273 (1882).
2. A. A. Ashcheulov, A. B. Belikov, and S. M. Kulikovskaya, Opt. Zh., No. 8, 68-70 (1993).
3. A. A. Ashcheulov, Opt.-Mekh. Prom., No. 12, 48-51 (1989).
4. A. A. Snarskii, Fiz. Tekh. Poluprovodn., **11**, Issue 10, 2053-2055 (1977).
5. V. N. Slipchenko, Ukr. Fiz. Zh., **21**, No. 1, 126-131 (1976).
6. V. N. Slipchenko, Some Problems of the Thermodynamics of an Anisotropic Thermoelement, Author's Abstract of Candidate's Thesis, Chernovtsy (1975).
7. L. D. Landau and E. M. Lifshits, Electrodynamics of Continua [in Russia], Moscow (1982).
8. A. A. Ashcheulov, I. V. Gutsul, and A. I. Rarenko, Ukr. Fiz. Zh., **38**, No. 6, 923-928 (1993).